BIBECHANA
Vol. 20, No. 3, December 2023, 259–266

ISSN 2091-0762 (Print), 2382-5340 (Online)
Journal homepage: http://nepjol.info/index.php/BIBECHANA
Publisher:Dept. of Phys., Mahendra Morang A. M. Campus (Tribhuvan University)Biratnagar

An accurate theoretical formula for linear momentum, force and kinetic energy

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Abstract

The paper demonstrates that the existing mathematical formulas of linear momentum, force and kinetic energy in physics are incomplete, since such formulas have been formulated without incorporation of mass-energy equivalence relation $E = mc^2$. Therefore, new reformulations of the main equations of linear momentum, force and kinetic energy in the realm of special relativity are proposed. The proposed formulas provide same mathematical outcomes as the old formulas, displaying same behavior of the system when velocity approaches to speed of light, but, most importantly, comprise only velocity of the light and mass of object to provide well-defined expressions. If c be speed of light in vacuum, then, the modified linear momentum, force and kinetic energy are given, respectively, by formulas $p = c\sqrt{m^2 - {m_o}^2}$, $F = \frac{cm}{\sqrt{m^2-{m_o}^2}.}\frac{dm}{dt}$ and $KE = \frac{c^2\left(m^2-m_0^2\right)}{2m}$, where m_o denotes the rest mass. These formulas vividly reveal that every physical variable depends solely on relativistic mass. Therefore, it modifies Newton's second law of motion and states that the force depends on rate of change of relativistic mass of object rather than its velocity. In this highly interesting topic, primary purpose here has been to present a succinct and the carefully reasoned account of a new aspect of the Newton's second law of motion which properly allows to derive the new mathematical formulas of linear momentum, force and kinetic energy.

Keywords
Force, kinetic energy, Linear momentum, Newton's second law of motion, Special theory of relativity.

Article information
Manuscript received: June 7, 2023; Accepted: September 16, 2023
DOI https://doi.org/10.3126/bibechana.v20i3.55476

1 Introduction

In 1687, Sir Isaac Newton presented three laws of motion in his seminal work "Principia Mathematica Philosophiae Naturalis," [1] which have produced most profound effect in entire field of physics to generate the mathematical expression of every dynamical variable such as linear momentum, force, kinetic energy and so on. Linear momentum is a fundamental parameter of physics which is employed to provide an accurate mathematical description of every physical phenomenon. There is hardly any field of theoretical physics or law of nature which is not associated with linear momentum. One of the most important applications of the momentum is in the formulation of Newtown's second law of motion [2] which states that the rate of change of linear momentum is equal to force. In mathematical form:

$$p = mv \ , \qquad F = \frac{d(p)}{dt} = \frac{d(mv)}{dt} \tag{1}$$

On the basis of Newtonian mechanics, the kinetic energy due to application of this force is given by,

$$KE = \frac{mv^2}{2} \tag{2}$$

The earliest known publication with the formula (1) is the work by Mach in 1883 in his system of mechanical definitions: moving force is the product of the mass-value of body into the acceleration inducted in that body [3]. A more extensive analysis of equations (1) and (2) can be found in work [4]. In Newtonian mechanics, equations (1) and (2) were stated with the fact assumption that mass m is constant. For over 200 years the equations of motion enunciated by Newton were believed to describe nature correctly and the first time that an error in these laws were discovered. It was discovered by Einstein in 1905. Einstein gave following famous equation called principle of mass and energy equivalence [5, 6].

$$E = mc^2 \tag{3}$$

Where m denotes the relativistic mass and E denotes the relativistic energy (total energy of body, means sum of kinetic and resting energy). In 1908, Lewis showed [7] that the principle of mass and energy equivalence equation (3) implies a relativistic mass formula of energy namely equation (4). The same derivation is given in Feynman lectures on physics [8]. In 1912, Tolman considered the principle of momentum conservation in a perfect inelastic collision and introduced theoretical dependence of mass from velocity based on the relativity principle [9].

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

Where m_0 denotes the rest mass. Derivations such as that by Tolman for relativistic mass are propagated in several excellent textbooks including the famous Feynman's lectures [10]. During the past few decades, many authors have written extensively on incorporation of relativistic formulas namely equations (3) and (4) to modify Newtonian equations (1) and (2). The article [11] presents the original definition of acceleration in the special theory of relativity while article [12] presents the formalism of relativistic acceleration and velocities in three dimensions of space. The work [13] presents the Einstein's mass-energy equivalence and the equation of relativistic mass, momentum and energy from the Newton's second law of motion. The article [14] presents mass-energy equivalence formula $E = mc^2$ from Maxwell's equations. Ref. [15] presents a relativistic paradox which exposes the true nature and ubiquity of hidden momentum. The work [16] develops an original derivation of Lorentz transformation in three-dimensional space, while work [17] shows the variation of mass in gravitational field with the use of formula $E = mc^2$. Articles [18], [19], [20] presents research on the special theory of relativity on De-Broglie wavelength of a particle and on electric permittivity and magnetic permeability of electromagnetic wave. There are numerous publications [21, 22] that examine the various aspects of the special theory of relativity. Hu [23] presented the derivation of the expression for the relativistic momentum and energy of relativistic particle based on relativistic addition. Sunego and Pin [24] presented a new derivation of the expression for momentum and energy of relativistic particle. Adkins [25] obtained the special relativistic expressions for momentum and energy in a totally inelastic variant of the Lewis-Tolman symmetric collision. There are many publications on special relativity but, most importantly, following questions have not yet addressed carefully. Above equation (3) suggests that the energy of a system depends solely on the inertial mass of system and velocity of light. The question then arises, "How do we express the mathematical formula of linear momentum, force and kinetic energy only in terms of mass and velocity of light as that of relativistic energy $E = mc^2$?". This paper provides the answer of this question by expressing the linear momentum, force and kinetic energy in terms of mass and velocity of light, then, in place of equations (1) and (2), we have simply,

$$\begin{aligned} p = c\sqrt{m^2 - {m_o}^2}, \ F = \frac{cm}{\sqrt{m^2 - {m_o}^2}} \frac{dm}{dt} \\ KE = \frac{c^2}{2m}\left(m^2 - m_0^2\right) \end{aligned} \tag{5}$$

Further, mass-energy equivalence principle namely equation (3) helps in transforming above equations into following form.

$$\begin{aligned} p = \frac{\sqrt{E^2 - E_0^2}}{c}, \quad F = \frac{E}{c\sqrt{E^2 - {E_o}^2}} \frac{dE}{dt}, \\ KE = \frac{E^2 - {E_o}^2}{2E} \end{aligned} \tag{6}$$

where E_o denotes the rest mass energy of system. It should be noted that for old theory namely equation (1) and (2), both variables mass m and velocity of object v are comprised, while for the modified theory (5) and (6), only one variable i.e., either mass or energy of system is comprised.

The structure of the remainder of this paper is organized as follows. In the section 2, dynamical variables such linear momentum, force, kinetic energy and de Broglie wavelength are expressed in terms of relativistic energy. In section 3, expression of linear momentum and force is transformed into modified form that involves single variable mass. Further, modified formulas are employed to derive famous mass-energy equation $E = mc^2$. Some conclusions are summed up in the last section.

2 Methods

2.1 Relativistic momentum and force in terms of energy

According to special theory of relativity, whenever an object of rest mass m_o is in speed, it seems to get heavier. The following equation gives the mass of object at travelling velocity v.

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{7}$$

or, $mc^2 = \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}}$

or, $E = \frac{E_o}{\sqrt{1-\frac{v^2}{c^2}}}$

Where $E_0 = m_0 c^2$ is the rest energy and $E = mc^2$ is the total energy possessed by the object. Squaring both sides of above equation we get,

$E^2 = \frac{{E_0}^2}{1-\frac{v^2}{c^2}}$

or, $1 - \frac{v^2}{c^2} = \frac{{E_0}^2}{E^2}$

or, $\frac{v^2}{c^2} = 1 - \frac{{E_0}^2}{E^2}$

$$\frac{v^2}{c^2} = \frac{E^2 - E_0^2}{E^2} \tag{8}$$

or, $\frac{v}{c} = \frac{\sqrt{E^2-E_0^2}}{E}$

or, $Ev = c\sqrt{E^2 - E_0^2}$

or, $mc^2 v = c\sqrt{E^2 - E_0^2}$

or, $mv = \frac{\sqrt{E^2-E_0^2}}{c}$

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} \tag{9}$$

where the product of mass and velocity $mv = p$ is the momentum of object. This equation (9) gives the relation between linear momentum p and relativistic energy E of moving object. Above relation suggests that linear momentum depends upon change in energy of the system $\sqrt{E^2 - E_0^2}$. Also, Newton's second law, in its most general form says that the rate of a change of a particle's linear momentum p is given by the force acting on the particle. In mathematical form:

$F = \frac{dp}{dt}$

Using equation (9) we get,

or, $F = \frac{d}{dt}\left(\frac{\sqrt{E^2-E_0^2}}{c}\right)$

or, $F = \frac{d\sqrt{E^2-E_0^2}}{cd(E^2-E_0^2)}\frac{d(E^2-{E_o}^2)}{dt}$

or, $F = \frac{1}{2c\sqrt{E^2-{E_o}^2}}\frac{2E\ \ dE}{dt}$

$$F = \frac{E}{c\sqrt{E^2 - {E_o}^2}}\frac{dE}{dt} \tag{10}$$

This result shows the expression of force in term of relativistic energy of system. In plain and simple terms, it means that force acting on a system depends upon change of energy due to relativistic phenomenon. Equation (9) plays an important role to show the relationship between de Broglie wavelength and energy of particle. De Broglie wavelength is an important concept while studying quantum mechanics. The wave length λ that is associated with an object in relation to its linear momentum and mass is known as de Broglie wavelength. A particle's de Broglie wavelength is usually inversely proportional to its linear momentum as follows.

$\lambda = \frac{h}{p}$

Using equation (9) we get,

$$\lambda = \frac{hc}{\sqrt{E^2 - {E_o}^2}} \tag{11}$$

Equation (11) shows the expression of de Broglie wavelength in terms of relativistic energy of the system. In plain and simple terms, it means that de Broglie wavelength of particle depends upon change of energy due to relativistic phenomenon. Further, the formula for kinetic energy KE for a particular body in terms of linear momentum is expressed as,

$$K.E. = \frac{p^2}{2m}$$

Using equation (9) we get,

$KE = \frac{1}{2m}\left(\frac{\sqrt{E^2-{E_o}^2}}{c}\right)^2$

or, $KE = \frac{E^2-{E_o}^2}{2mc^2}$ (12)

$$KE = \frac{(E^2 - {E_o}^2)mc^2}{2(mc^2)^2} \tag{12}$$

Total energy of body is $E = mc^2$

Hence, $KE = \frac{(E^2-{E_o}^2)mc^2}{2E^2}$

or, $KE = \frac{mc^2}{2}(1 - \frac{E^2}{E_0^2})$

From equation (8) we have,

$KE = \frac{mc^2}{2}\frac{v^2}{c^2}$

$KE = \frac{mv^2}{2}$

Above derivation suggests at once that modified formula of linear momentum namely equation (9) is completely true, since it gives correct expression of kinetic energy KE. Therefore, modified expression of force namely equation (10) is also true. Equation (10) is the basic law of physics on which the relation between relativistic energy and force of a system is based.

Rewriting equation (12) we have,

Kinetic energy $KE = \frac{E^2 - {E_o}^2}{2mc^2}$

Substituting $mc^2 = E$ we get,

Kinetic energy

$$KE = \frac{E^2 - {E_o}^2}{2E} \tag{13}$$

Therefore, the kinetic energy associated with body can be accurately determined by knowing only relativistic energy.

2.2 Relativistic momentum and force in terms of mass

In special relativity, the relativistic mass is given by,

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

where c is speed of light in vacuum and m denotes the mass of body when it is moving with a velocity v. Then total relativistic energy of the body of mass m is given by,

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

The momentum of the body $p = mv$ so that $v = \frac{p}{m}$

Hence, $mc^2 = \frac{m_o c^2}{\sqrt{1-\frac{p^2}{m^2c^2}}}$

or, $mc^2 = \frac{m_o c^2}{\sqrt{1-\frac{p^2c^2}{m^2c^4}}}$

or, $m^2c^4 = \frac{m_0^2c^4}{1-\frac{p^2c^2}{m^2c^4}}$

or, $m^2c^4(1 - \frac{p^2c^2}{m^2c^4}) = {m_o}^2c^4$

or, $m^2c^4 - p^2c^2 = {m_o}^2c^4$

or, $p^2c^2 = m^2c^4 - {m_o}^2c^4$

$$p^2 = \left(m^2 - {m_o}^2 \right) c^2 \tag{14}$$

A particle of mass m moving with a velocity v has a wave associated with it whose wavelength according to De-Broglie is given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$p = \frac{h}{\lambda}$$

Squaring both sides,

$$\frac{h^2}{\lambda^2} = p^2 \tag{15}$$

From equation (14) and (15), we have

$$\frac{h^2}{\lambda^2} = \left(m^2 - {m_o}^2 \right) c^2$$

or, $\lambda^2 = \frac{h^2}{c^2\left(m^2 - {m_o}^2 \right)}$

$$\lambda = \frac{h}{c\sqrt{m^2 - {m_o}^2}} \tag{16}$$

This is the expression for De-Broglie wavelength due to variation of mass with velocity. Thus, equation (16) reveals the dependence of wave nature of object with relativistic phenomenon. This shows that the De-Broglie wavelength associated with particle exists whenever mass of particle varies with velocity.

Further, the relationship between kinetic energy and momentum is given by

$KE = P^2 2m$

Using equation (14)

$$KE = \frac{c^2}{2m} \left(m^2 - m_0^2 \right) \tag{17}$$

From relativistic variation of mass with velocity we have,

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Squaring both sides of above equation we get,

or, $m^2 = \frac{{m_0}^2}{1-\frac{v^2}{c^2}}$

or, $1 - \frac{v^2}{c^2} = \frac{{m_0}^2}{m^2}$

or, $\frac{v^2}{c^2} = 1 - \frac{{m_0}^2}{m^2}$

or, $\frac{v^2}{c^2} = \frac{m^2 - m_0^2}{m^2}$

or, $m^2 - m_0^2 = \frac{m^2v^2}{c^2}$

Above equation (17) becomes,

Kinetic energy $KE = \frac{c^2}{2m} \frac{m^2v^2}{c^2}$

Kinetic energy $KE = \frac{mv^2}{2}$

Above derivation suggests at once that modified formula of momentum namely equation (14) is completely true, since it generates correct formula of kinetic energy. From equation (17) it is seen that kinetic energy completely depends on change of mass of body due to relativistic phenomenon.

Kinetic energy $KE = \frac{c^2}{2m} \left(m^2 - m_0^2 \right)$

Therefore, the kinetic energy associated with body can be accurately determined by knowing only relativistic mass of body.

3 Results and Discussion

According to Einstein, the mass of the body in motion is different from the mass of the body at rest.

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

This is the relativistic formula for variation of mass with velocity. Where m_0 denotes the rest mass and m denotes the relativistic mass of body. Squaring both sides of above equation,

or, $m^2 = \frac{{m_0}^2}{1-\frac{v^2}{c^2}}$

or, $1 - \frac{v^2}{c^2} = \frac{{m_0}^2}{m^2}$

$$v = c\frac{\sqrt{m^2 - {m_o}^2}}{m} \tag{18}$$

or, $mv = c\sqrt{m^2 - {m_o}^2}$

or, $p = c\sqrt{m^2 - {m_o}^2}$

The most important consequence of this equation is that linear momentum actually depends upon the change of relativistic mass.

From Newton's second law of motion,

$F = \frac{dp}{dt} = \frac{d(mv)}{dt}$

or, $F = m\frac{dv}{dt} + v\frac{dm}{dt}$

From equations (18) we get,

or, $F = m\frac{d}{dt}\left(c\sqrt{1 - \frac{{m_0}^2}{m^2}}\right) + c\sqrt{1 - \frac{{m_0}^2}{m^2}}\frac{dm}{dt}$

or, $F = \frac{cm}{2\sqrt{1 - \frac{{m_0}^2}{m^2}}}\frac{d}{dt}\left(1 - \frac{{m_0}^2}{m^2}\right) + c\sqrt{1 - \frac{{m_0}^2}{m^2}}\frac{dm}{dt}$

or, $F = \frac{cm}{2\sqrt{1 - \frac{{m_0}^2}{m^2}}}\frac{2{m_0}^2}{m^3}\frac{dm}{dt} + c\sqrt{1 - \frac{{m_0}^2}{m^2}}\frac{dm}{dt}$

or, $F = \left(\frac{m_0^2}{m^2\sqrt{1 - \frac{{m_0}^2}{m^2}}} + \sqrt{1 - \frac{{m_0}^2}{m^2}}\right)c\frac{dm}{dt}$

or, $F = \left(\frac{m_0^2}{m\sqrt{m^2 - {m_o}^2}} + \frac{\sqrt{m^2 - {m_o}^2}}{m}\right)c\frac{dm}{dt}$

or, $F = \left(\frac{m_0^2 + m^2 - {m_o}^2}{m\sqrt{m^2 - {m_o}^2}}\right)c\frac{dm}{dt}$

or, $F = \frac{m^2}{m\sqrt{m^2 - {m_o}^2}}c\frac{dm}{dt}$

$$F = \frac{cm}{\sqrt{m^2 - {m_o}^2}}\frac{dm}{dt} \tag{19}$$

Again, the total relativistic energy associated with mass m is given by,

$E = mc^2 = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

or, $mc^2 = \frac{m_o c^2}{\sqrt{1 - \frac{m^2 v^2}{m^2 c^2}}}$

or, $m^2 c^4 (1 - \frac{m^2 v^2}{m^2 c^2}) = {m_o}^2 c^4$

or, $m^2 c^4 - m^2 v^2 c^2 = {m_o}^2 c^4$

or, $m^2 v^2 c^2 = m^2 c^4 - {m_o}^2 c^4$

or, $m^2 v^2 = \left(m^2 - {m_o}^2\right) c^2$

The momentum of a body is $p = mv$.

Hence, $p^2 = \left(m^2 - {m_o}^2\right) c^2$

$$p = c\sqrt{m^2 - {m_o}^2} \tag{20}$$

Since c is a constant, momentum p depends only on the mass of system.

$p \; \alpha \; \sqrt{m^2 - {m_o}^2}$

Above equation (20) is a fundamental formula of linear momentum that does not involve velocity of body. Thus, linear momentum is always related with relativistic mass variation rather than its velocity. There is immense application of modified formula (20) as compare to old formula $p = mv$. Since, old formula of linear momentum involves two variables m and v. Therefore, it is very difficult to find explicit relation between momentum and mass due to involvement of another variable velocity v, but modified formula (20) involves only a single variable mass m. As a result, it is easy to determine the clear relationship between momentum and mass of object. Rewriting equation (20),

$p = c\sqrt{m^2 - {m_o}^2}$ Comparing this equation with linear equation $y = \mu x + C, we\ get$

$y = p$, $\mu = c$, $x = \sqrt{m^2 - {m_o}^2}$, $C = o$

It is interesting and instructive to sketch the graph between momentum p and $\sqrt{m^2 - {m_o}^2}$ (mass of body) taking them along Y-axis and X-axis respectively. The slope of graph gives a constant c which is the speed of light in free space as shown in figure (1).

The modified formula of linear momentum has huge application in physics to perceive important ground breaking results such as $E = mc^2$. It generates new expression of force that involves single variable mass of body and excludes its velocity. Let a force F act upon the body in the direction of its motion. Force is the rate of change of momentum p i.e.

$F = \frac{dp}{dt}$

From modified formula of linear momentum namely equation (20),

or, $F = \frac{d}{dt}(c\sqrt{m^2 - {m_o}^2})$

or, $F = \frac{cd\sqrt{m^2 - m_0^2}}{d(m^2 - m_0^2)}\frac{d(m^2 - {m_o}^2)}{dt}$

or, $F = \frac{c}{2\sqrt{m^2 - {m_o}^2}}.\frac{2m \; dm}{dt}$

$$F = \frac{cm}{\sqrt{m^2 - {m_o}^2}}\frac{dm}{dt} \tag{21}$$

The meaning of this equation is that force F upon the body explicitly depends on the rate of change of mass of object rather than the change of velocity of body. Thus, this formula of force namely (21) corresponds to Einstein's mass energy formula $E = mc^2$because both force and energy depends only upon the mass of the body rather than velocity of body. If ds be the displacement of the body due to the force, then work done by the force is given by,

$dw = F ds$

substituting value of F from equation (21),

$dw = \frac{cm}{\sqrt{m^2 - {m_o}^2}}\frac{dm}{dt}ds$

The velocity of body $v = \frac{ds}{dt}$

Hence,

$$dw = \frac{cmv}{\sqrt{m^2 - {m_o}^2}}dm \tag{22}$$

Also, from special relativity,

$$\frac{c}{m} = \frac{v}{\sqrt{m^2 - {m_o}^2}} \tag{23}$$

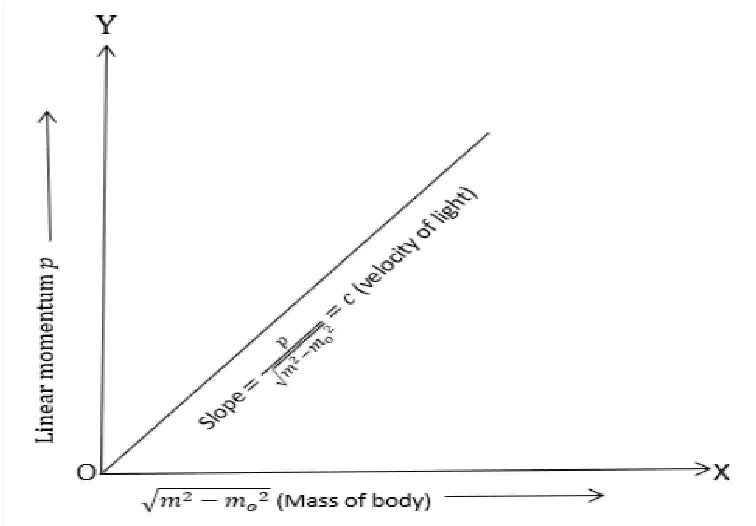

Figure 1: Linear relation between momentum and mass of body.

From equation (22) and (23) we get,

$dw = \frac{vc^2}{v}$dm

or, $dw = c^2 dm$

When mass of the body changes from m_o and m, then total work done is given by,

$\int_0^w dw = c^2 \int_{m_o}^m dm$

or, $w = (m - m_o)c^2$

or, $mc^2 = w + m_o c^2$

where $m_o c^2$ is the energy due to rest mass of the body i.e., it's energy when at rest with respect to the observer is called it rest energy $m_o c^2$. Similarly, mc^2 is the total energy E possessed by the body. Then we have,

Total energy (E) = Work done (w) + Rest energy (E_o)

$$E = (m - m_o)c^2 + m_o c^2$$

$$E = mc^2$$

This is known as Einstein's mass energy relation. Therefore, the modified formula of force given by equation (21) is completely true because it gives exactly same as Einstein mass energy equation.

It is interesting and instructive to compare old formulas and modified formulas of variables such as linear momentum, force, kinetic energy and wavelength. In old formulas, dynamical variables are expressed in terms of mass m and velocity v of object as shown in table (1). Exactly opposite behavior occurs when these dynamical variables are modified by using relativistic mechanics. As shown in table (1), formulas of dynamical variables in modified form depend on velocity of light c instead of velocity of object v.

Table 1: Formulas of dynamical variables.

S.N.	Dynamical Variables	Formulas		
		Old formula	Modified formula in terms of Mass	Modified formula in terms of Energy
1	Linear momentum	$p=mv$	$p=c\sqrt{m^2-m_o^2}$	$p=\frac{\sqrt{E^2-E_0^2}}{c}$
2	Force	$F=\frac{d(mv)}{dt}$	$F=\frac{cm}{\sqrt{m^2-m_o^2}}\frac{dm}{dt}$	$F=\frac{E}{c\sqrt{E^2-E_o^2}}\frac{dE}{dt}$
3	Wave length	$\lambda=\frac{h}{mv}$	$\lambda=\frac{h}{c\sqrt{m^2-m_o^2}}$	$\lambda=\frac{hc}{\sqrt{E^2-E_o^2}}$
4	Kinetic Energy	$KE=\frac{mv^2}{2}$	$KE=\frac{c^2}{2m}(m^2-m_0^2)$	$KE=\frac{E^2-E_o^2}{2E}$

4 Conclusion

All possible relativistic formulas of dynamical variables such as linear momentum p, force F, kinetic energy KE and de Broglie wavelength λ have been thoroughly derived in this article. The key innovated formulas of dynamical variables in terms of relativistic energy can be written from equations (9), (10), (11) and (13) as follows.

$$p = \frac{\sqrt{E^2 - E_0^2}}{c}\ ,\ F = \frac{E}{c\sqrt{E^2 - {E_o}^2}}\frac{dE}{dt}$$

$$\lambda = \frac{hc}{\sqrt{E^2 - {E_o}^2}}\ , KE = \frac{E^2 - {E_o}^2}{2E}$$

According to mass-energy principle, mass may appear as energy and energy as mass. Therefore, above formulas of dynamical variables can be written in terms of mass from equations (16), (17), (19) and (20) as follows.

$$p = c\sqrt{m^2 - {m_o}^2}\ , F = \frac{cm}{\sqrt{m^2 - {m_o}^2}}\frac{dm}{dt}$$

$$\lambda = \frac{h}{c\sqrt{m^2 - {m_o}^2}}\ , KE = \frac{c^2}{2m}\left(m^2 - m_0^2\right)$$

The relativistic energy of system $E = mc^2$ depends on inertial mass of system. In same way, it is concluded that every physical variable depends solely on relativistic mass as shown in above equations. The most dramatic success of this paper is the modification of Newton's second law of motion which states that force F depends on rate of change of mass rather than change of velocity with time. The derivation of well- known formula for energy $E = mc^2$ has been derived by using modified equation of force. The primary purpose here has been to provide the possible extension of special relativity to modify the Newtonian formulas of dynamical variables and lay down the basic equations of extended theories. The new formulas elucidated here will have many physical applications and it will be of interest in many other areas of theoretical physics.

Conflict of Interest: The author declares no conflict of interest.

Data Availability: Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

List of symbols

F: Force

p: Linear momentum

KE: Kinetic energy

m_0: Rest mass

m: Relativistic mass

c: Velocity of light

v: Velocity of body

E: Relativistic energy

E_0: Rest mass energy

λ: Wavelength

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