Bulletin of the Department of Geology

Bulletin of the Department of Geology, Tribhuvan University, Kathmandu, Nepal, Vol. 12, 2009, pp. 89–94

Dynamic modelling in slopes using finite difference program

Subodh Dhakal

Department of Geology, Tri-Chandra Campus, Tribhuvan University, Nepal

ABSTRACT

The finite difference program FLAC is used for dynamic modelling of slopes whereby the relationship of the slope geometry, earthquake input signals (mainly frequency of the wave) and the material properties with amplification of vibration on the surface are investigated. At the same time, the influence of varying input frequencies is also investigated. The higher slopes were amplified most by the lower input frequency whereas the reverse was true for the smaller slopes. The overall magnitude of the amplification was maximum with input signals of higher frequency and lower slope heights. The horizontal amplification as much as 17 (horizontal acceleration in the order of 1.7g) was obtained for the normal limestone slope with 20 m height when an input signal of 15 Hz frequency was applied. This experiment revealed that for extremely lower values of shear modulus, there was mostly attenuation instead of amplification and for extremely high values of shear modulus, amplification was negligible as compared to the certain range of intermediate shear modulus. Maximum amplification in the order of 6.5 (horizontal acceleration of 0.66g) was achieved for the shear modulus of 3000 MPa where slope height was of 40 m.

INTRODUCTION

Ground response analyses are most important to predict the ground surface motions during dynamic loading and to evaluate the earthquake induced forces. Numbers of techniques have been developed so far for the ground response analysis in terms of one, two and three dimensional ground response. It is not always possible to arrive at an analytical solution for the problem such as stress strain analysis (Desai and Abel 1972). Numerical modelling technique is one of the convenient ways of such analysis. Ground response analysis and the topographical effect on ground response are investigated by numbers of authors in the past. They all agreed that topography and site conditions played much significant role in amplifying the ground (Boore 1972; Davis and West 1973; Bouchon 1973; Geli et al 1988). Castro (1999) estimated a concentration of damage towards the slope edges in a range of about 40 m from the edges for the earthquake of 1999 in Columbia. Under such circumstances, finite difference numerical modelling program FLAC (Fast Langrangian Analysis of continua) was used for this research to investigate the response of ground slopes during dynamic loading to understand the change in amplification due to the change in properties of the ground. Shear modulus, frequency and wavelength of the input signals and the height of slope were varied to investigate the relation of those parameters with the amplification.

*Corresponding author:

E-mail address: subodhdhakal@hotmail.com

NUMERICAL MODELLING PROGRAM

FLAC (Fast Langrangian analysis of continua) is finite difference numerical program. It was originally developed by Peter Cundall (Cundall, 1976) and commercially released by ITASCA, C.G., Inc. Capacity of performing large two dimensional calculations without excessive memory requirements and the facility for dynamic modelling makes it versatile in rock mechanics and Earthquake Engineering. In FLAC, every derivative in the set of governing equations is replaced directly by an algebraic expression written in terms of the field variables (like stress or displacement) at discrete points in space. Dynamic modelling can be executed in FLAC using either of two methods: Equivalent Linear Method or Fully Non-Linear method. Equivalent linear method is used in this study which was defined by Seed and Idriss (ITASCA 2000). In this method a linear analysis is performed with same initial values assumed for damping ratio and shear modulus.

Numerical Simulation for dynamic loading

Numerical calculation for dynamic loading for the slope with simple geometry is carried out to investigate the influence of slope geometry and material properties on the amplification during dynamic loading by earthquake. The histories of the displacement, velocity and acceleration are studied in some of the selected zone of the slope. The concentration of the study is towards the crest of the slope as the literatures are saying those parts are amplified more than the other parts. For the simulation purpose, some of the simple schematic slope profiles are generated but it is tried to make the section more close to reality. The dimension of the model is kept small to moderate in order to reduce the calculation time and the memory needed for the computer. An example of slope geometry used for this study is shown in Fig 1. Like in other finite element modelling programs, in FLAC, the structure is built out of finite amount of elements by discretization resulting in a mesh.

Model Formulation

The stress levels in this study are such that failure of intact material is expected, and most of the geological materials show linear elastic and perfectly plastic brittle weakening behaviour. Such type of stress strain relationship is more close to reality and can be accurately modelled by Mohr-Coulomb model. Due to all of these reasons, Mohr-Coulomb plasticity model was used in this study. Bulk modulus, shear modulus, density, friction angle, cohesion and tensile strength are needed as material properties in Mohr-Coulomb constitutive model. The detail of the material properties used for this study is given in Table 1. The slope angle was fixed at 25.5° whereas slope height, shear modulus and bulk modulus were variable since their relations with amplification were to be determined.

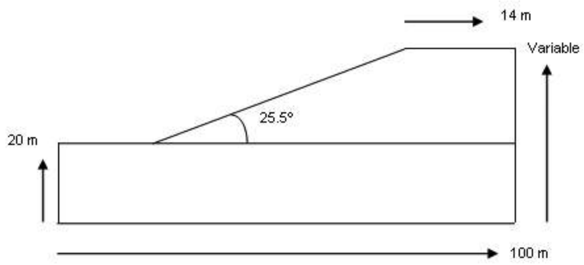

Fig. 1 General geometry of the slope model used for the numerical calculation

Regarding the boundary condition, it was differentiated for static and dynamic stage calculation. Static calculation is done prior to the dynamic analysis. For the static analysis, first and the last column in the x-direction and first row in the y-direction were fixed. This helped to prevent the model to move as a body and generate lateral stresses during the consolidation process. For the dynamic calculation however, free-field boundaries were applied to the two vertical sides of the model to avoid distortions in the wave transmissions and to make the model close to reality since the slopes were not isolated with the other topographical features in the lateral sides. The base was kept rigid with the introduction of quiet boundaries at the first row in x- and y-direction. In this study, a sinusoidal wave function with acceleration of 1 m/s^2 and duration of 0.25 sec. was defined using the FISH function available in FLAC. The frequency of the input signals however was varied to investigate its effect on amplification. Later on the defined input function was applied through the bottom of the model. For the dynamic analysis, Rayleigh damping was applied to compensate the energy dissipation through the medium. Rayleigh damping of 5% was used for

Table 1: Geotechnical material properties and input wave frequency used for the simulation

Density (Kg/m^3)	Friction angle (?)	Cohesion (Mpa)	Tensile strength (Mpa)	Input frequency (Hz)
2000	42	6.7	1.58	5

Table 2: Material properties used in the study to find the relation of different parameters with amplification

Parameter to find	Slope height	Density (Kg/m^3)	Shear Modulus	Bulk Modulus
Shear modulus versus amplification	Variable	2000	Variable	200 MPa
Slope height, frequency versus amplification	Variable	2000	10000 Mpa	200 MPa
Bulk modulus versus amplification	Variable	2000	10 Mpa	Variable

this numerical dynamic analysis as the damping for the geological materials lies between 2 and 5%.

RESULTS AND DISCUSSIONS

Shear modulus versus amplification

Ground motion caused by earthquakes is generally characterized in terms of ground surface displacement, velocity and acceleration. Geotechnical engineers use acceleration rather than other two because acceleration is directly related to the dynamic forces that earthquakes induce on the soil or rock mass (Day 2001). The amplification in this study refers to the ratio of maximum output acceleration in the crest of the slope along the horizontal direction to the input acceleration because it was found that in all the slope models the crest of the slopes were highly amplified compared to the other parts. The input parameters used for the numerical calculation by varying shear modulus is listed in Table 2. Input frequency was fixed at 5 Hz and the parameters slope angle, friction angle, tensile strength and cohesion were fixed as Simulation in FLAC reveals that amplitude of wave was increased for the material with lower shear modulus and decreased for the higher values. However, for the extremely lower values of shear modulus there was mostly attenuation instead of amplification. In addition, the harmonic effect strongly controls the amplification of the slope due to the change in shear modulus. The peaks of amplification in Fig. 2 are the result of the resonance effect in particular combination of material properties and input signals. In this study, the maximum amplitude of amplification on varying shear modulus was 6.5 for the slope height of 40 m with the material having shear modulus of 3000 MPa.

Slope height and input frequency versus amplification

The input parameters used for the numerical calculation to find the relation of slope height and input frequency with amplification is listed in Table 2, however the slope height and frequency was varied continuously to find their relation with amplification. Other material properties were same as that shown in Table 1. Similarly, bulk modulus was also varied but for the analysis purpose, the value of bulk modulus that gives maximum amplification was considered. Present study revealed that amplification of slope was strongly frequency dependent as some slopes were amplified more by one input frequency and other slopes with different heights & material properties were amplified more by another input frequency.

As an example, for the input frequency of 10 Hz, a 40 m slope gave the maximum amplification, whereas for the input frequency of 15 Hz, a 20 m slope gave maximum amplification (Fig. 3). It is noteworthy to mention here that the amplification as much as 17 (horizontal acceleration in the order of 1.7 g) was obtained for the slope height of 20 m due to input frequency of 15 Hz. The result agreed in general that the taller objects would have the greater influence of lower frequencies whereas shorter objects were more affected by the higher frequency signals, because taller objects would have smaller resonance frequency than the shorter. In addition, harmonic effect was strongly reflected as it could be seen in the plot of the slope height versus maximum amplification with different input frequencies (Fig. 3). Generation of standing waves by harmonic frequencies that controlled the amplification of the slope might be the reason behind it. In addition, the peaks of amplification curve shifted towards lower slope heights on increasing the input frequency as shown by Dhakal et al, 2004.

Bulk modulus versus amplification

The material properties used for finding the relation of bulk modulus with amplification are given in Table 2. Material properties other than those mentioned in Table 2 are shown in Table 1. The calculation was

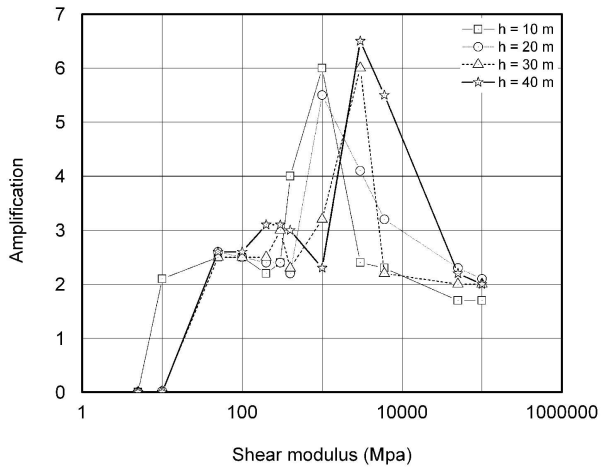

Fig. 2 Shear modulus versus amplification for different slope heights (h). The slope angle for all the cases is 25.5° and wave frequency is 5 Hz

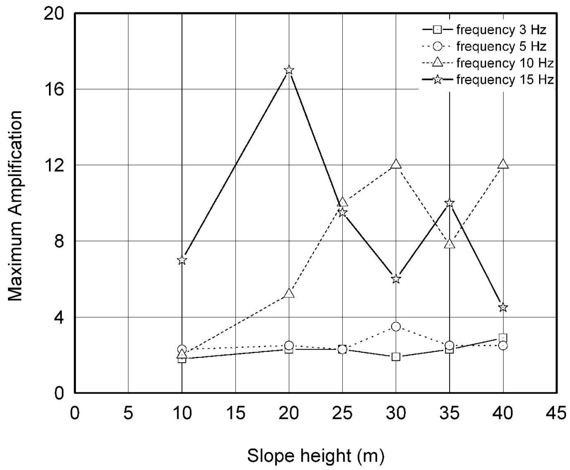

Fig. 3 Variation of amplification with slope height (h) for different input frequencies with slope angle of 25.5°

performed for slope heights of 10 m, 20 m, 25 m, 30 m, 35 m, and 40 m. For all those sets of slope height and bulk modulus combination, calculation was repeated for the input frequencies of 3 Hz, 5 Hz, 10 Hz, and 15 Hz.

One of the sample plots of the bulk modulus versus amplification is shown in Fig. 4. The general trend of the curve is similar for all the combinations of input frequency and slope height. There are clearly two threshold values of bulk modulus from where the amplification either starts to decrease or remains constant for some particular value of bulk modulus. In general, the first threshold value range from 500-1000 MPa from where the amplification starts to decrease. For the higher bulk modulus, amplification begins to decrease approximately linearly on a log scale until for about 5000 MPa giving a clear slope in the curve afterwhich amplification remains constant giving a second threshold value. It is not fully understood why the amplifications for the certain range of bulk modulus values remains constant. The function for the bulk modulus versus amplification can be given by the equations 1, 2 and 3.

If $Log(K) < Log(K_1)$, then $A = A_{max}$ (1)

If $Log(K_1) < Log(K) < Log(K_2)$, (2)

If $Log(K) > Log(K_2)$, $A = A_{min}$ (3)

where, K is input bulk modulus, K_1 is first threshold bulk modulus, K_2 is second threshold bulk modulus (Fig. 5), A is any amplification, A_{max} is maximum amplification and A_{min} is minimum amplification for the particular combination of slope height and frequency.

Slope height/wavelength ratio versus amplification

The relation between the slope height and the wavelength of the input signals with the amplification will have great importance in the engineering design of the slope in earthquake prone areas. For the design earthquake, the wavelength cannot be changed whereas the engineering slope height can be designed according to need. It is remarkable to notice in Fig. 6 that there is highest amplification for the slope height to wavelength ratio of 0.07 to 0.20 clearly shown by the first big peak (position of peaks is variable for different slope heights) in the graph. Horizontal amplification as much as 6.5 is observed for the slope height of 40 m with the slope height to wavelength ratio of 0.17. There is another smaller peak afterwards and then the amplification diminishes. Probably the standing waves are generated when the input wave is reflected back from the slope crest and interfere with the upcoming wave from below the crest, resulting constructive interference.

Application of the results

Although this study is model based, most of the

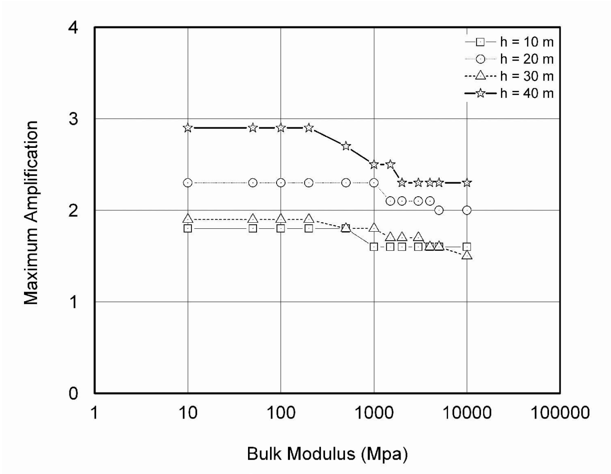

Fig. 4 Bulk modulus versus maximum amplifications for different slope heights (h). The input frequency in this figure is 15 Hz and slope angle is 25.5°

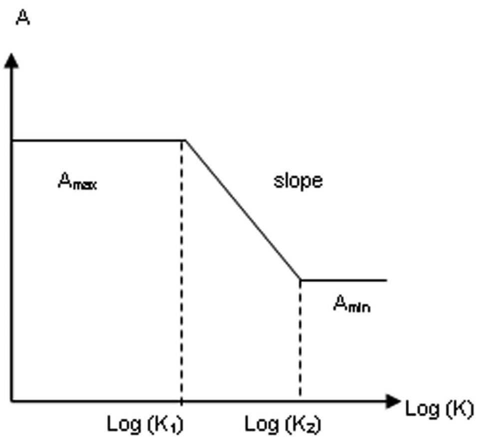

Fig. 5 Schematic diagram showing general shape of the function for bulk modulus versus amplification

geotechnical parameters used in the simulation are based on laboratory results for some of the limestone, which are useful in the slopes of Nepal as well. The range of input frequency, slope angle and the slope height are also within the practically possible limit. In the situation of no prior knowledge about the actual input earthquake signal, the results are to be estimated for the designed earthquake. In the context of cut and fill method of road construction and other infrastructure development in Nepal, engineering design of slopes mostly in terms of angle and height is common. Results of this study indicate the possibility of disastrous slope amplifications in the case of adverse combinations of slope geometry, input earthquake signals and material properties. One can not influence the earthquake magnitude and the waves generated from it, however the effects from earthquake event can be minimized to great extent if the results of the studies like this are considered during the planning of infrastructure development. If the wave frequency and amplitude of the possible earthquake for a particular area is known, the slopes can be designed so that the natural frequency of the slope does not coincide with that of the generated earthquake frequency. Therefore, slope geometry, geotechnical material properties and probable earthquake wave parameters should be considered together during the design of the slopes and to minimize the effect of earthquake in natural slopes as well.

Comparisons with past studies

This study agrees qualitatively with many of the previous studies related with the topography effect of seismic amplification. However, because of the difference in the input parameters and slope geometries, quantitative comparisons with the past studies were not possible. The crests of the slopes are found to amplify high compared to the flank similar to the results obtained by Boore (1972) and Davis and West (1973). Results coincide qualitatively with the findings of Athanasopoulos et al. (1998) that the concentration of damage due to the 1995 earthquake around the town of Egion, Greece was in the elevated region adjacent to the crest of high and steep escarpment. Boore (1972) also found that the seismic amplification was strongly frequency dependent which is evident in this study as well (Fig. 3). The results also agree qualitatively with that of Davis and West (1973) where they found the resonance effect for the amplification and that it depends on the material properties and wavelength of the input frequency. Present study also depicts the resonance and harmonic effects as shown in Figs. 2 and 3.

CONCLUSIONS

It is found that the amplification is strongly frequency dependent and harmonic effect strongly controls the magnitude and repetition of maximum

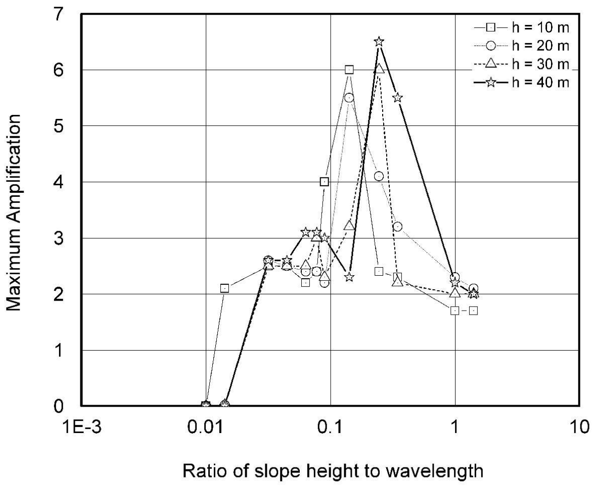

Fig. 6 Ratio of slope height to wavelength versus amplification for different slope heights (h). The slope angle for this calculation is 25.5° and input frequency is 5 Hz.

amplification. Extremely lower shear modulus brings maximum attenuation and extremely higher values of shear modulus resist the wave to amplify. Amplification fluctuates to great extent for the intermediate values of shear modulus and it is dependent strongly on the resonance frequency of the slope and harmonic effect. Regarding the slope height, shorter slopes are amplified more by the application of higher input frequency whereas taller slopes are amplified more by smaller input frequencies. Maximum horizontal amplification of 17 (horizontal acceleration of 1.7g) as obtained in this study indicate the probable scale of disaster if the material properties, slope geometry and earthquake input signals used for this simulation match in reality. At the same time, the slope height to wavelength ratio of 0.07 to 0.23 is found extremely vulnerable for amplification. Such type of relationship and probable scale of problem should be considered during the design of slope. The earthquake prone countries in general, and developing countries with high probability of earthquake in particular, should be aware about this fact and should be considered during the design of slope to prevent the damage due to anticipated earthquake in that particular area.

ACKNOWLEDGMENT

The author is thankful to International Institute for Geo-information Science and Earth Observation (ITC), The Netherlands for giving opportunity to conduct this research. Delft University of Technology, The Netherlands is also acknowledged for giving work place and providing necessary equipments during research period. The author also extends kind appreciation to Dr. Robert Hack and Dr. Marco Huisman at ITC, The Netherlands.

REFERENCES

Athanasopoulos, G. A., Pelekis, P. C., and Leonidou, E. A., 1998. Effects of surface topography and soil conditions on the seismic ground response-including liquefaction in the Egion (Greece) 15/6/1995 earthquake. In: Proceedings of the Eleventh European Conferences on Earthquake Engineering, Abstract volume, pp. 525–525.

Boore, D. M., 1972. A note on the effect of simple topography on seismic SH waves. Bulletin of Seismological Society of America, v. 62, pp. 275–284.

Bouchon, M., 1973. Effect of topography on surface motion. Bulletin of Seismological Society of America, v. 63 (3), pp. 615–632.

Castro, E., 1999. Topographic site characterstics and damage pattern of the January 25th 1999 earthquake in Armenia-Colombia, The Netherlands. M.Sc. Thesis, ITC Delft, pp 84.

Cundall, P., 1976. Explicit Finite Difference Methods in Geomechanics in Numerical Methods in Engineering. Proceedings of the EF Conference on Numerical Methods in Geomechanics, Blacksburg, Virginia, v. 1, pp. 132–150.

Davis, L. L. and West, L. R., 1973. Observed effects of topography on ground motion. Bulletin of Seismological Society of America, v. 63, No. 1, pp. 283–298.

Day, R., 2001. Geotechnical Earthquake Engineering Handbook. New York, McGraw-Hill, 612 pp.

Desai, C. S., and Abel, J. F., 1972. Introduction to the Finite Element Method: A Numerical Method for Engineering Analysis, Van Nostrand Reinhold Company, New York.

Dhakal, S., Hack, R., and Huismann, M., 2004. Earthquake-induced slope vibration and its relationship with material properties, slope height and input wave frequency. Journal of Nepal Geological Society, v. 30, pp. 135–140.

Geli, L., Bard, P. Y., and Jullien, B., 1988. The effect of topography on earthquake ground motion: A review and new results. Bulletin of Seismological Society of America, v. 78 (1): pp. 42–63.

ITASCA, 2000. FLAC 4.0 manuals, Minnessotta, ITASCA Consulting Group, Inc.