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ON Φ-RECURRENT LORENTZIAN α-SASAKIAN MANIFOLD WITH SEMI SYMMETRIC NON METRIC CONNECTION

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Abstract

The present work deals with the study of Φ-recurrent Lorentzian α-Sasakian manifold with semi-symmetric non metric connection.

Keywords and Phrases

Locally Φ-symmetric manifold Φ-recurrent Lorentzian α-Sasakian manifold; **η-** *Einstein manifold*

Introduction

The notion of local symmetry of a Riemannian manifold has been studied by many authors in several ways to a different extent. In 1977, Taka hasi [9] introduced the notion of locally Φ-symmetric Sasakian manifold and obtained their several interesting results. Generalizing the notion of Φ-symmetry, De, U.C [4] introduced the notion of Φ-recurrent Sasakian manifold.

Fridmann and Schouten introduced the idea of semi-symmetric linear connection on a differentiable manifold. Hayden introduced the idea of metric connection with torsion on Riemannian manifold. Yano [8], Golab [5] defined and studied semi-symmetric and quarter symmetric connection with affine connection. Further many authors like De, U.C. [1], Sharfudin and Hussain [3], Rastogi, Mishra and Pandey, Bagewadi and many others studied the various properties of semi-symmetric connection.

In this paper we study Φ-recurrent Lorentzian α-Sasakian manifold with semi-symmetric non metric connection and proved that a Φ-recurrent Lorentzian α-Sasakian manifold with symmetric non metric connection is a **η-** Einstein manifold. Further we show that in Φ-recurrent Lorentzian α-Sasakian manifold with semi-symmetric non metric connection, the characteristic vector ξ and vector field **η** associated to the 1- form A are co-directional.

Preliminaries: A differentiable manifold M of dimension n is called a Lorentzian α sasakian manifold of it admints a tensor field Φ of type (1, 1), the characteristic vector ξ, a covariant vector field η and lorentzian metric g which satisfy

$\Phi^2 = 1 + \eta \otimes \xi$ (2.1)

$\eta(\xi) = -1$ (2.2)

$g(\Phi X, \Phi Y) = g(X,Y) + \eta(X)\,\eta(Y)$ (2.3)

$g(X, \xi) = \eta(X)$ (2.4)

$\Phi\,\xi = 0,\ \eta(\Phi X) = 0$ (2.5)

$(D_x\Phi)Y = \alpha\, g(X, Y)\,\xi - \alpha\,\eta(Y)\,X$ (2.6)

For all X, Y ∈ Tm [2, 3, 13]

Also a lorentzian α sasakian manifold m satisfies

$(D_x\xi)Y = \alpha\,\Phi\,X$ (2.7)

$(D_x\eta)Y = -\alpha\, g(\Phi X,Y)$ (2.8)

Where D denotes the operator of covariant differentiation with respect to lorentzian matric g.

Also on a Lorentzian α sasakian manifold, the following hold [2, 3, 13]

$R(X, Y)\,\xi = \alpha^2\,(\eta(Y)\,X - \eta(X)\,Y)$ (2.9)

$R(\xi, X)\,Y = \alpha^2(g(X, Y)\,\xi - \eta(Y)\,X)$ (2.10)

$R(\xi, X)\,\xi = \alpha^2\,(\eta(X)\,\xi + X)$ (2.11)

$S(X, \xi) = (n-1)\,\alpha^2\,\eta(X)$ (2.12)

$\eta(R(X, Y)Z) = \alpha^2(g(Y, Z)\,\eta(X) - g(X, Z)\,\eta(Y))$ (2.13)

$g(R(\xi, X)Y, \xi) = -\ \alpha^2\,[g(X,Y) + \eta(X)\,\eta(Y)]$ (2.14)

For any vector field X, Y, Z where S is the Ricci curvature and Q is the Ricci operation given by

$S(X, Y) = g\,(\Phi X, Y)$

A lorentzian α sasakian manifold is said to be η- Einstein manifold if its Ricci tensor S takes the form

$S(X, Y) = a\, g(X, Y) + b\, \eta(X)\, \eta(Y)$

For arbitrary vector X, Y where a and b are function on M. If b=0 the η- Einstein manifold becomes Einstein manifold. [3, 9] have proved that if Lorentzian α sasakian manifold M is η- Einstein manifold then $a + b = -\alpha^2(n-1)$.

Definition 2.1: A Lorentzian α sasakain manifold is said to be locally Φ- symmetric if

$$\Phi^2((D_w R)(X, Y) Z) = 0 \qquad (2.15)$$

Definition: 2.2

A Lorentzian α sasakian manifold is said to be recurrent if there exists a non zero 1-form A such that

$$\Phi^2((D_w R)(X, Y) Z) = A(W) R(X, Y) Z, \qquad (2.16)$$

Where A (W) is defined by A (W) =g(W, ρ) and ρ is a vector field associated with 1- from.

Lorentzian α sasakian manifold with semi symmetric non metric connection:

A semi symmetric connection $\bar{D}$ in Lorentzian α sasakian manifold can be defined by

$$\bar{D}_X Y = D_X Y + \eta(Y) X \qquad (3.1)$$

Also we have $(\bar{D}_X g)(Y,Z) = -\eta(Y) g(Y,Z) - \eta(Z) g(Y,X)$ (3.2)

A connection given by (3.1) with (3.2) is called semi symmetric non metric connection in Lorentzain α sasakian manifold.

A relation between curvature tensor M of the manifold with semi metric connection non metric connection $\bar{D}$ and Levi- Civita connection D is given by

$$\bar{R}(X, Y)Z = R(X, Y) Z - \alpha\, g(\Phi X, Z)Y - \alpha\, g(\Phi Y, Z)X \qquad (3.3)$$

Where $\bar{R}$ and R are the Riemannian curvature of the connections $\bar{D}$ and D respectively.

From (3.3), we have $\bar{S}(Y, Z) = S(Y, Z) + \alpha(n-1)\, g(\Phi Y, Z)$ (3.4)

Where $\bar{S}$ and S are the Ricci tensor of the connections $\bar{D}$ and D respectively.

Contracting (3.4), we get $\bar{r} = r$ (3.5)

Where $\bar{r}$ and r are the scalar curvatures of the connections $\bar{D}$ and D respectively.

Φ- recurrent Lorentzian α sasakian manifold with semi symmetric non metric connection.

A analogous to the definition (2.2) we define a Lorentzian α sasakian manifold is said to be Φ - recurrent with respect to semi symmetric non metric connection if its curvature tensor $\bar{R}$ satifies the following condition

$$\Phi^2 (D_w \bar{R}) (X, Y) Z) = A(W) \bar{R}A(W) \bar{R}(X, Y) Z \quad (4.1)$$

Using (2.1) in (4.1), we get

$$(\bar{D}_w\bar{R}) (X, Y) Z + \eta (((\bar{D}_w\bar{R})(X,Y)Z)\xi = A(W) \bar{R}(X, Y)Z \quad (4.2)$$

From which it follows that

$$g((\bar{D}_w\bar{R})(X, Y)Z, U) + \eta(((\bar{D}_w\bar{R})(X,Y)Z)g (\xi, U) = A(W)g(\bar{R}(X, Y)Z, U) \quad (4.3)$$

Let $\{e_i\}$, i = 1, 2, 3,...... n be an orthonormal basis of the tangent space at any point of the manifold. Then putting X = U =$\{e_i\}$ in (4.3) and taking summation over i, $1 \leq i \leq n$, we get

$$(\bar{D}_w\bar{S}) (Y, Z) + \eta(((\bar{D}_w\bar{R})(e_i,Y) Z) \eta(e_i) = A(W) \bar{S}(Y, Z) \quad (4.4)$$

Putting Z = ξ, in (4.4), the second term of (4.4) takes the form

$g(((\bar{D}_w\bar{R})(e_i, Y) \xi , \xi)$ which on simplification gives $g(((\bar{D}_w\bar{R})(e_i, Y) \xi , \xi) = 0$

Then from (4.4) we obtain

$$(\bar{D}_w\bar{S}) (Y, \xi) = A (W) \bar{S}(Y, \xi) \quad (4.5)$$

Now we know that

$$(\bar{D}_w\bar{S}) (Y, \xi) = \bar{D}_w\bar{S}(Y, \xi) - \bar{S}(\bar{D}_w Y, \xi) - \bar{S} (Y, \bar{D}_w \xi) \quad (4.6)$$

Using (2.7), (2.8), (2.12), (3.4) in (4.6), we get

$$(\bar{D}_w\bar{S}) (Y, \xi) = \alpha S(Y, \Phi W) + S(Y, W) - \alpha(\alpha + 1) (n-1) g(Y, \Phi W) - \alpha^2(n -1) g(Y,W) + \alpha^2(n -1) g(\Phi Y, \Phi W) \quad (4.7)$$

In view of (4.5) and (4.7), we get

$$\alpha S (Y, \Phi W) + S(Y, W) - \alpha(\alpha + 1)(n - 1)g(Y, \Phi W) - \alpha^2(n - 1)g(Y, W) + \alpha^2(n -1) g(\Phi Y, \Phi W) = \alpha^2(n -1) A(W) \eta(Y)$$

Replacing Y = ΦY in above equation, we get

$\alpha S(\Phi Y, \Phi W) + S(\Phi Y, W) - \alpha(\alpha + 1)(n - 1)g(\Phi Y, \Phi W) - \alpha^2(n - 1)g(\Phi Y, W) + \alpha^2(n - 1) g(Y, \Phi W) = 0$ (4.8)

Interchanging Y and W in (4.8) we get

$\alpha S(\Phi W, \Phi Y) + S(\Phi W, Y) - \alpha(\alpha + 1)(n - 1)g(\Phi W, \Phi Y) - \alpha^2(n - 1)g(\Phi W, Y) + \alpha^2(n - 1) g(W, \Phi Y) = 0$ (4.9)

Adding (4.8) and (4.9) and simplifying we get

$$S(\Phi Y, \Phi W) = (\alpha^2 + 1)(n-1)g(\Phi Y, \Phi W)$$

Using (2.3) and (2.15), we get

$$S(Y, W) = (\alpha^2 + 1)(n-1)g(Y, W) + (n - 1)\,\eta(Y)\,\eta(W)$$

This leads to the following theorem.

Theorem 4.1: A Φ - recurrent Lorentzian α sasakian manifold with semi symmetric non metric connection is η- Einstein manifold.

Again from (4.2), we have

$(\bar{D}_W\bar{R})(X, Y)Z = -\eta(((\bar{D}_W\bar{R})(X, Y)Z)\,\xi + A(W)\,\bar{R}(X, Y)$ (4.10)

From (2.13), (3.3) and using Bainchi identity we get

$A(W)\,\eta(\bar{R})(X, Y)Z) + A(X)\,\eta(\bar{R}(Y, W)Z) + A(Y)\,\eta(\bar{R}(W, X)Z) = 0$ (4.11)

From (2.13), (3.3) in (4.11) we get

$A(W)\,\alpha^2[g(Y, Z)\,\eta(X) - g(X, Z)\,\eta(Y)] + A(X)\,\alpha^2[g(Z, W)\,\eta(Y) - g(Y, Z)\,\eta(W)] + A(W))\,\alpha^2[g(X,W)\,\eta(Z) - g(Z, W)\,\eta(X)] + \alpha[g(\Phi Y, Z)\,\eta(X) - g(\Phi X, Z)\,\eta(Y) + g(\Phi W, Z)\,\eta(Y) - g(\Phi Y, Z)\,\eta(W) + g(\Phi X, Z)\,\eta(W) - g(\Phi W, Z)\,\eta(X)] = 0$ (4.12)

Putting Y = Z = e_i in (4.12) and taking summation over i, $1 \leq i \leq n$, we get

$A(W)\,\eta(X) = A(X)\,\eta(W)$ (4.13)

For all vector fields, W. Replacing X by ξ in (4.13), we get

$$A(W) = -\eta(\rho)\,\eta(W) \quad (4.14)$$

For any vector field W, where $A(\xi) = g(\xi, \rho) = \eta(\rho)$, ρ being vector field associated to the

1-form A that is $g(X, \rho) = A(X)$

From (4.13) and (4.14) we state that following.

Theorem 4.2: In a Φ- recurrent Lorentzian α sasakian manifold with semi symmetric non metric connection the characteristic vector ξ and vector field ρ associated to the 1- form A are codirectional and 1- form A is given by (4.14).

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