**ISSN: 2469-9062 (print), 2467-9240(e)**

*Journal of Institute of Science and Technology, 2015, **20(2):** 80-83, © IOST, Tribhuvan University*

# Inverse Gaussian Model to Describe the Distribution of Age Specific Fertility Rates of Nepal

**Arjun Kumar Gaire[1] and Roshan Aryal[2]**

*[1]Department of Science and Humanities*
*Khwopa Engineering College, Bhaktapur, Nepal*
*[2]Maharajgunj Medicine Campus, Institute of Medicine, Tribhuvan University, Kathmandu, Nepal*
*E-mail: arjun.gaire@gmail.com*

**ABSTRACT**

This paper attempts to formulate Inverse Gaussian probability model to describe the distribution of the age specific fertility rates among Nepalese mothers. The average age of child bearing has been found to be 26.12 years whereas child bearing starts at the age of 15.82 years. Estimated total fertility rate (TFR) has been found to be 2.60 against the observed value of 2.6 per woman. The coefficient of determination and chi-square values suggest that the model found to be good fit to the Nepal Demographic and Health Surveys data. Findings may help researchers and policy makers for applying the model in order to obtain the descriptive statistics for policy interventions.

**Keywords:** Chi-square test, coefficient of determination, parameters, total fertility rate, formulation of model

## INTRODUCTION

The fertility is one of the key demographic indicators like migration and mortality. The age specific fertility rates (ASFR) measure the annual number of children birth to women of a specific age or age group. The modeling of ASFRs in case of Asian countries has been used with some limitations (Islam 2011), however, modeling of fertility data has been paid great attention among the demographers around the world.

A number of researchers have tried to develop several models in order to know the distributional patterns of fertility and fertility related parameters around the globe (Brass 1974, 1978, Coale & Trussell 1974, Mitra 1967, Islam & Mallick 1987, Hoem *et al.* 1981, Gilje 1969). The models like Polynomial models (Brass 1978), mixture Hadwigr function (Chandola *et al.* 1999), the quadratic Spline function (Schmertmann 2003) were used to describe the distributional patterns of fertility performance of the population. Peristera and Kostaki (2007) developed six parameters model while Mazzuco and Scarpa (2011) developed the skew-symmetric probability density function to describe the distribution of the fertility performance.

Brass (1974, 1978) developed a linear relationship between a double logarithmic transformation of the age patterns as well as the standard age schedules. However, the limitation of this model is the lack of interpretations of the demographic parameters (Beer 2011).

A mixture Hadwiger model with seven parameters was applied by Chandola *et al.* (1999). A number of models have been developed to study fertility performance (Aryal 2005, 2011, Aryal & Paudel 2012, Brass 1974, 1978). There were several limitations that the discussed models were not good fit to the data of bi-modality and uni-modality (Beer 2011). The skew-symmetric probability density function was applied for this purpose by Mazzuco and Scarpa (2011). The piece-wise quadratic splines function proposed by Schmertmann (2003) with 13 parameters was found good fit to the wide variety of fertility data, however, it does not include all parameters, lack of interpretations and applications for further fertility projection (Beer 2011).

In this context, the Inverse Gaussian Probability Distribution (IGPD) model has been applied to describe the distribution of ASFR pattern among Nepalese mothers, which is considered as the function of age of women of child bearing age of 15-49 years. The IGPD model has only three parameters ($\alpha$, $\mu$ and $\lambda$). Demographic interpretations of these parameters are: $\alpha$ is the threshold parameter, ($\mu + \alpha$) is the mean age of child bearing among mothers, and $\lambda$ is a scale parameter of the model. For this purpose, the Nepal Demographic and Health Surveys (MOH 2007, 2012) data are used in order to check the suitability and validity of the used model, and the chi-square test as well as the coefficient of determination are also used to check the fitting of the used model.

## MATERIALS AND METHODS

### Inverse Gaussian Probability Model

Let X denotes the age of mother at the birth of child bearing and ASFR is a functi3 of age of mother at the birth of child bearing. Such a function is described by IGPD with three parameters and it is given below:

$$f(x) = \sqrt{\frac{\lambda}{2\pi(x-\alpha)^3}} exp\left\{\frac{-\lambda(x-\alpha-\mu)^2}{(x-\alpha)\mu^2}\right\} \quad (1)$$

where, $x > \alpha$ is the threshold parameter, $(\mu + \alpha)$ is the average of the distribution and $\lambda > 0$ is a scale parameter of the model. The variance of the age of mother is $\frac{\mu^3}{\lambda}$.

### Estimation of Parameters

Three parameters of the IGPD are calculated by using method of maximum likelihood estimation (MLE). The literature of which is available on Mahmoud (1991), the Bayesian estimation of the three parameters of IGPD. To estimate the parameters of IGPD, the Easy Fit 5.6 software for distribution fitting has been used.

### Model Validity

To test the accuracy of the model, the chi-square test of goodness of fit is used. Also, the coefficient of determination between observed and theoretical ASFRs has been obtained. The chi-square statistic is calculated as:

$$\chi^2 = \sum_{i=1}^{n}\left\{\frac{(O_i - E_i)^2}{E_i}\right\} \quad (2)$$

where $_i$ is the observed ASFRs and $E_i$ is the expected or theoretical ASFRs obtained from the fitting of IGPD for Nepalese mothers.

## RESULTS

The IGPD is applied to the data of ASFRs of Nepalese mothers using the Nepal Demographic and Health Surveys. Three parameters of the IGDP are obtained by using the Easy Fit 5.6 Software for distribution fitting and presented in the Tables 1 and 2.

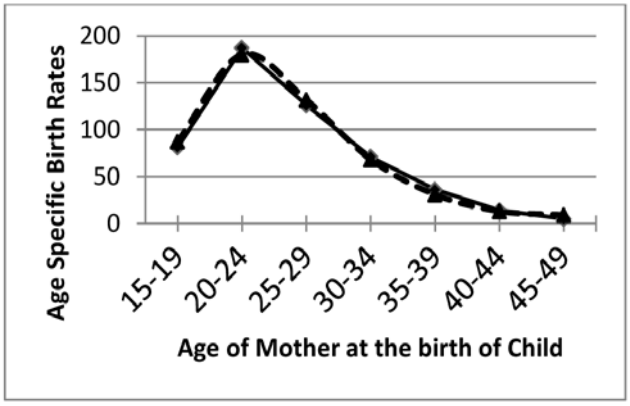
Model estimates the threshold parameter ($\alpha$) of 15.82 years, which implies that the age of child bearing for Nepalese mothers starts at the age of 15.82 years based on the data of 2011. The average age of child bearing has been found to be 26.12 years with mode of the distribution of 22.31 years. This result indicates that the fertility performance among Nepalese mothers has been found highest at the age group of 20– 24 years. The estimated TFR has been found to be 2.6, which is closed to the observed value of TFR of 2.6 per woman indicating that IGPD has been found good fit to the data of Nepal. The value of coefficient of determination between observed and theoretical value of ASFRs obtained from IGPD has been found to be 0.99, which shows that the observed value of ASFRs has been explained 99 percent by the model.

**Table 1. Observed and expected distribution of ASFRs of NDHS (2011) data by using IGPD**

| Age group (in years) | Observed ASFRs | F(x) = P(X ≤ x) | Expected ASFRs |
|---|---|---|---|
| 15 – 20 | 81 | 0.1678 | 87.24 |
| 20 – 25 | 187 | 0.5139 | 179.97 |
| 25 – 30 | 126 | 0.7671 | 131.66 |
| 30 – 35 | 71 | 0.8982 | 68.18 |
| 35 – 40 | 36 | 0.9574 | 30.81 |
| 40 – 45 | 14 | 0.9826 | 13.09 |
| 45 – 50 | 5 | 0.9930 | 9.04 |
| Total | 520 | - | 520 |
| TFR | 2.60 | - | 2.60 |
| Chi square | | 3.258 | |
| d.f. | | 3 | |
| α | | 15.816 | |
| μ | | 10.3 | |
| λ | | 85.041 | |
| $R^2$ | | 0.9933 | |

*The dotted line is for the ASFRs obtained by IGDP and Solid line is for observed value.*

**Fig. 1. Observed and expected distribution of ASFRs of NDHS-2011**




The Fig. 1 shows that the fertility performance among Nepalese mothers starts at the age of 15.82 years with increases exponentially and gains peaked at the age group of 20–24 years and decreases then thereafter it and attains least values at the age group of 45–49 years.

**Table 2. Observed and expected distribution of ASFRs of NDHS-2006 data by using IGPD**

| Age group (in years) | Observed ASFRs | $F(x) = P(X \leq x)$ | Expected ASFRs |
|---|---|---|---|
| 15 – 209 | 98 | 0.1703 | 106.41 |
| 20 – 25 | 234 | 0.5219 | 219.80 |
| 25 – 30 | 144 | 0.7758 | 158.65 |
| 30 – 35 | 84 | 0.9045 | 80.43 |
| 35 – 40 | 48 | 0.9612 | 35.43 |
| 40 – 45 | 16 | 0.9846 | 14.64 |
| 45 – 50 | 2 | 0.9940 | 9.64 |
| Total | 625 | | 625 |
| TFR | 3.1 | | 3.13 |
| | Chi square | 13.728 | |
| | d.f. | 3 | |
| | α | 15.686 | |
| | μ | 10.265 | |
| | λ | 86.869 | |
| | $R^2$ | 0.9816 | |

Model estimates the threshold parameter (α) of 15.69 years, which depicts that age of child bearing starts at the age of 15.69 years. The average age of child bearing among mothers has been found to be 25.91 years. The mode of the distribution has been found to be 22.23 years, which means that the fertility performance among Nepalese mothers has been found highest at the age group of 20-24 years. The estimated TFR has been found to be 3.13 per woman while the observed value of 3.1 per woman. The result indicates that IGPD model has been found good fit to the data of 2006. The value of coefficient of determination between observed and expected value of ASFRs obtained from IGPD has been found to be 0.98, which shows that the observed value of ASFRs has been explained 98.16 percent by the model.

*The dotted line is for the ASFRs obtained by IGPD and Solid line is for observed value.*

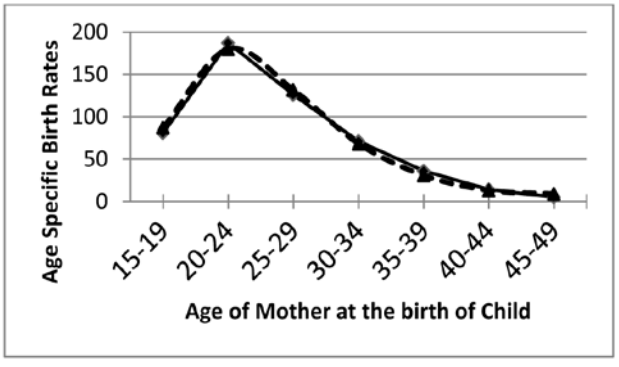



**Fig. 2. Observed and expected distribution of ASFRs of NDHS-2006 data**

The Fig. 2 shows that the fertility performance among Nepalese mothers starts at the age of 15.69 years with increases exponentially and gains peaked at the age group of 20-24 years and decreases then thereafter and attains least at the age group of 45 – 49 years.

## CONCLUSION

The IGPD has been applied to describe the distribution of ASFRs of Nepal using NDHS data. The model has been found to be good fit to the data of Nepal. The model suggests that the child bearing age starts at the age of 15.82 years while the average age of child bearing has been found to be 26.12 years. Estimated TFR through model has been found to be 2.60 against the observed value of 2.6 per woman by using the data of 2011. The value of the coefficient of determination ($R^2$) between observed and expected values of ASFRs has been found to be 0.99, which shows that the observed ASFRs has been explained 99.33 percent by the estimated ASFRs indicating the model has been found good fit to the data of Nepal. Similarly, the child bearing age starts at the age of 15.69 years, the average age of child bearing has been found to be 25.91 years and TFR by the model has been found to be 3.13 against observed value of 3.1 per woman of the 2006 data. The value of coefficient of determination ($R^2$) between observed and estimated ASFRs has been found to be 0.98, which shows that the observed ASFRs explains 98.16 percent by the estimated ASFRs indicating that the model has been found good fit to the data of 2006 as well. Findings of this paper may help researchers and policy makers for applying the model in order to obtain the descriptive statistics for policy interventions.

## REFERENCES

Aryal, T.R. 2005. A new technique to etimate fertility from birth order statistics. *Perspectives on Higher Education: Journal of the University Campus* **2&3:** 57-61.

Aryal, T.R., Paudel, C.M. 2012. *Demographic models for describing the tempo in fertility*. Tribhuvan University Bulletin Special Edition.

Aryal, T.R. 2011. *Fertility Dynamics of Nepal.* Ekta Book Distributors, Kathmandu, Nepal.

Brass, W. 1974. Perspectives in Population Prediction: Illustrated by the statistics of England and Wales. *Journal of the Royal Statistical Society Series* A **137:** 532-583.

Brass, W. 1978. The relational Gompertz model of fertility by age of women. London school of hygiene and tropical medicine (mimeographed).

Beer, J.D. 2011. A new relational method for smoothing and projecting age specific fertility rates (TOPALS). *Demographic research* **24** (Article 18) *:* 409-454.

Chandola, T.D., Coleman, D.A., Homs, R.W. 1999. Recent European fertility patterns: fitting curves to distorted distributions. *Population Studies* **53** : 317–329.

Coale, A.J. and Trussell, T.J. 1974. Model fertility schedules:. variation in the age structure of childbearing in human populaitons. *Populaiton Index* **40** :185-258.

Gilje, E. 1972. *Analytic graduation of age specific fertility rates*. Oslo , Norway.

Hoem, J.M., Madsen, D., Nielsen, J.L.,Ohlsen, E.M, Hensen, H.D., Rennermalm, B. 1981. Experiments in modeling recent Danish fertility curves. *Demography***18**: 231-244.

Islam, M. 2011. Modeling of age specific fertility rates of Jakarta in Indonesia; a polynomial model approach. *International Journal of Science and Engineering Research* **2** (iissue 11).

Islam, N.M., Mallick, S.A. 1987. On the use of a truncated Pearsonian type III curve in fertility estimation. *Dhaka University Studies Part B Science* **35(1)**: 23-32.

Mazzuco, S., Scarpa, B. 2011. Fitting age specific fertility rates by a skew-symmetrical probability density function. University of Padova, Working paper series, No 10, Italy.

Mitra, S., Romaniuk, A. 1973. Pearsonian type I curve and its fertility projection.

MOH. 2007. Nepal Demographic and Health Survey Report -2006. Ministry of Health, New ERA and Macro International Inc. Kathmandu , Nepal

MOH. 2012. Nepal Demographic and Health Survey Report 2011, Population Division Ministry of Health and Population, Government of Nepal, Kathmandu.

Peristera, P., Kostaki, A. 2007. Modeling fertility in modern populations. *Demographic Research* **16***(* Article 6): 141-194.

Schmertmann, C.P. 2003. A system of model fertility schedules with graphically intuitive parameters. *Demographic research* **9(**Article 5): 81-110.